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REDUCTION OF THE BAROMETER

TO SEA LEVEL.

WITH TABLES.

BY CHARLES CARPMAEL, M.A., F.R.A.S.,

(LATE FELLOW OF ST. JOHN'S COLL., CAMB.)

Deputy Superintendent of the Meteorological Service of Canada.

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REDUCTION OF THE BAROMETER TO SEA LEVEL.

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The application of an approximately correct reduction to barometric readings, taken at various levels, in order to reduce them to what they would have been at one specified level, is absolutely necessary for their intercomparison. In the following paper several formulæ which have been employed for this purpose are examined; and tables are appended by means of which, with very little calculation, a sufficiently correct reduction may be obtained, and which are, moreover, peculiarly adapted to the computation of tables of reduction for individual stations.

Guyot's Tables* D, XVI. and XIX'., are commonly employed, on this continent, for the purpose of effecting the reduction. These give the height, in English feet, of a column of air corresponding to a tenth of an inch in the barometer at various temperatures, the barometric pressure at the base of the column being from 22 inches to 30·4 inches.

A formula is given for use with Table XVI., which may be written

$$R = \frac{Z}{N} \times \frac{\beta}{10\,b}, \qquad \text{(i.)}$$

where R represents the required reduction in inches, Z the difference of height between the two stations, or the height above the sea (expressed in feet), N the number in the table, β the observed reading of the barometer reduced to 32° Fahr., and b the pressure on which the tabular number N is based,† that is, 30 inches.

* Meteorological and Physical Tables. Third edition. Washington, 1859. By Arnold Guyot, P.D., LL.D., Professor of Geology and Physical Geography, College of New Jersey.

† Guyot defines what is here represented by b, as "the normal height of barometer at the sea-level," and in an example which he gives, he employs 30 in. It is, however, only because the table is based on a barometric reading of 30 in., that this value of b is to be employed.

No formula is given for use with Table XIX.′, but it is stated that the table may be employed "for reducing barometrical observations to the level of the sea, and also to any other level by a similar process." An example is, however, given, applying tables in French measure, corresponding to XIX.′, the method of which example may be represented by the formula

$$R = \frac{2\,Z}{{}_{\beta}N_{t} + {}_{B}N_{T}} \cdot \frac{1}{10}, \qquad \text{(ii.)}$$

where ${}_{\beta}N_{t}$ is the number in the table corresponding to the barometric reading* and temperature at the upper station, and ${}_{B}N_{T}$ that corresponding to those at the lower station; an approximate reduced barometric reading and temperature being employed in taking out the latter quantity.

Formula (i.) may also be employed with Table XIX.′, b being any height and N the number in the table corresponding to b. No advantage is, however, gained, by using this table instead of Table XVI. with formula (i.), unless b be taken nearly equal to β, so that we may have, nearly

$$R = \frac{Z}{10\,N}.$$

Laplace's formula for computing differences of elevation from barometrical observations, from which each of the above is deduced, may be written

$$Z = A_{t} \log \frac{B}{\beta}, \qquad \text{(iii.)}$$

where A_{t} is a constant, depending on the mean between the temperatures at the upper and lower stations. Strictly, it also depends upon the latitude of the station, and on the height above the sea; but the variations due to these may be neglected, unless the height is very considerable.

Now the number ${}_{b}N_{t}$, in the above mentioned tables, for barometer reading b, and temperature t, is the difference of elevation

*Throughout this paper, when a barometric reading is spoken of, the reading reduced to temp. 32° Fahr. is to be understood.

of two stations, the temperature being t, the barometer reading at lower station b, and at the upper station $b - \frac{1}{10}$. Hence, by (iii.),

$$ {}_bN_t = A_t \log \frac{b}{b - \frac{1}{10}} . $$

Also R being the reduction, (iii.) may be written

$$ Z = A_t \log \frac{\beta + R}{\beta} . $$

Combining these, we get

$$ \log \left(1 + \frac{R}{\beta}\right) = \frac{Z}{{}_bN_t} \log \frac{10b}{10b - 1} ; $$

hence,
$$ 1 + \frac{R}{\beta} = \left(\frac{10b}{10b - 1}\right)^{\frac{Z}{{}_bN_t}} = \left(1 - \frac{1}{10b}\right)^{-\frac{Z}{{}_bN_t}} $$

$$ = 1 + \frac{Z}{{}_bN_t} \cdot \frac{1}{10b} + \frac{1}{1.2} \cdot \frac{Z}{{}_bN_t} \cdot \overline{\frac{Z}{{}_bN_t} + 1} \cdot \overline{\frac{1}{10b}}\Big|^2 + \ldots . $$

by the binomial theorem.

$$ \therefore R = \beta \left(\frac{Z}{{}_bN_t} \cdot \frac{1}{10b} + \frac{1}{1.2} \cdot \frac{Z}{{}_bN_t} \cdot \overline{\frac{Z}{{}_bN_t} + 1} \cdot \overline{\frac{1}{10b}}\Big|^2 + \ldots . \right) \text{(iv.)} $$

Formula (i.) is deduced from (iv.), by neglecting all terms beyond the first; and making $b = 30$ inches, if used with Table XVI.; but, if used with Table XIX.', b may be any reading within the range of the table, and ${}_bN_t$ the *corresponding* number from the table.

Although (i.) is sufficiently accurate for small heights, it is evident, on comparing it with the full formula (iv.), that it becomes more and more inaccurate as the height increases.

If, in (i.), the reduced height B, were substituted for the observed height β, the error would be relatively less; for Laplace's formula may also be expanded in the form

$$ R = B \left(\frac{Z}{{}_bN_t} \cdot \frac{1}{10b} - \frac{1}{1.2} \cdot \frac{Z}{{}_bN_t} \cdot \overline{\frac{Z}{{}_bN_t} - 1} \cdot \overline{\frac{1}{10b}}\Big|^2 + \ldots . \right) \text{(v.)} $$

In this formula each term, after the first, is relatively smaller than the corresponding term in (iv.); and if $\frac{Z}{{}_bN_t}$ is large, the terms having sensible magnitude, are alternately positive and negative. Therefore the error, introduced by neglecting all terms beyond the first, is relatively less in (v.) than in (iv.); but, since B is not known until R has been determined, this formula could only be employed by successive approximation, and is therefore inconvenient.

It may be seen by inspection that, in Table XIX.', ${}_BN_t$ is very nearly equal to $\frac{\beta}{B} \cdot {}_\beta N_t$. That this should be so, may be proved thus :—

as already explained

$${}_BN_t = A_t \log \frac{10\,B}{10\,B - 1},$$

$${}_\beta N_t = A_t \log \frac{10\,\beta}{10\,\beta - 1}.$$

$$\therefore \frac{{}_BN_t}{{}_\beta N_t} = \frac{\log \frac{10\,B}{10\,B - 1}}{\log \frac{10\,\beta}{10\,\beta - 1}} = \frac{\log\left(1 - \frac{1}{10\,B}\right)}{\log\left(1 - \frac{1}{10\,\beta}\right)}.$$

$$= \frac{\frac{1}{10\,B} + \frac{1}{2} \cdot \overline{\frac{1}{10\,B}}\Big|^2 + \ldots.}{\frac{1}{10\,\beta} + \frac{1}{2} \cdot \overline{\frac{1}{10\,\beta}}\Big|^2 + \ldots.}.$$

$$= \frac{\beta}{B} \text{ nearly,} \qquad \text{(vi.)}$$

$$\therefore {}_BN_t = \frac{\beta}{B} \cdot {}_\beta N_t \text{ nearly, as above stated.}$$

From (iv.) and (v.), together with (vi.), we may deduce (ii.), thus: In (iv.), let $b = \beta$, we obtain

$$R = \frac{Z}{{}_{\beta}N_t} \cdot \frac{1}{10} + \frac{1}{1.2} \cdot \frac{Z}{{}_{\beta}N_t} \cdot \overline{\frac{Z}{{}_{\beta}N_t} + 1} \cdot \frac{1}{100\,\beta} + \cdots$$

$$\therefore\ {}_{\beta}N_t \cdot R = \frac{Z}{10} + \frac{1}{1.2} \cdot \frac{Z}{10} \cdot \overline{\frac{Z}{{}_{\beta}N_t} + 1} \cdot \frac{1}{10\,\beta} + \cdots\cdot$$

Similarly from (v.) making $b = B$,

$${}_{B}N_t \cdot R = \frac{Z}{10} - \frac{1}{1.2} \cdot \frac{Z}{10} \cdot \overline{\frac{Z}{{}_{B}N_t} - 1} \cdot \frac{1}{10\,B} + \cdots\cdot$$

$$\therefore\ \left({}_{\beta}N_t + {}_{B}N_t\right) R = \frac{2Z}{10} + \frac{1}{1.2} \cdot \frac{Z}{10} \cdot \left(\frac{Z}{10\beta.\,{}_{\beta}N_t} - \frac{Z}{10B.\,{}_{B}N_t} + \frac{1}{10\beta} + \frac{1}{10B}\right) + \cdots$$

But from (vi.) $B.\ {}_{B}N_t = \beta.\ {}_{\beta}N_t$ nearly.

$$\therefore\ \left({}_{\beta}N_t + {}_{B}N_t\right) R = \frac{2Z}{10} + \frac{1}{1.2} \cdot \frac{Z}{10} \cdot \left(\frac{1}{10\,\beta} + \frac{1}{10\,B}\right) \text{ nearly,}$$

or, neglecting the second term on the right,

$$R = \frac{2\,Z}{{}_{\beta}N_t + {}_{B}N_t} \cdot \frac{1}{10} \text{ nearly}$$

Here t is the mean between the temperatures at the upper and lower stations; whilst in (ii.) these two temperatures are respectively employed, in taking out the two numbers. The difference thus introduced is very trifling; as may easily be seen, if the value given below for A_t, be substituted in the expression for N_t.

Formula (ii.), like (v.), is objectionable, in that it assumes a knowledge of the reduced reading, which it is the object to ascertain.

The foregoing formulæ being all either inconvenient, or not sufficiently accurate except for small elevations, I have formed the accompanying tables (A and B), to facilitate the calculation of the reduction.

It will be noticed from the form of (iii.) that, at any place, the temperature being constant, the reduced reading, and therefore *also the reduction, varies as* β. It is, therefore, sufficient to calculate the reduction ${}_Z N_t$, for one barometer reading (b) only; from which that for any other reading may be obtained by a simple proportion. It is immaterial whether the value adopted for b be one which could be attained, or not; it may therefore be chosen with reference to convenience alone. In Table A, b is taken equal to 100 inches, so that the reduction for any reading (β) of the barometer, may be obtained by the formula

$$R = \frac{\beta}{100} \cdot {}_Z N_t .$$

Table A was calculated by means of formula (iii.), the value of A_t being taken as* $60345.51 \left(1 + \frac{t - 32}{450}\right)$. In this table is given the quantity ${}_Z N_t$, for values of Z equal to 100, 200, 300, &c. feet, for every second degree of temperature from $-40°$ to $100°$ Fahr., and also, the difference for the next 100 feet at each height. It is sufficient to employ first differences only, in using the table.

Table B is intended to diminish the labour in applying formula (iii.), as will be explained in the sequel.

Since calculating these tables, my attention has been called to a paper by Lieut. H. H. C. Dunwoody, U. S. Army, in the Report of the Chief Signal Officer, Washington, 1876. In this paper tables are given, based in part on observations taken by direction of the Chief Signal Officer, U. S. A., on Mount Washington, Mount Mitchell, and Pikes Peak.

In the first table is given the decrease of temperature for each 100 feet of elevation at each hour in the day. In the second table is given the "weight of a column of air 100 feet high, at different barometric pressures and temperatures, expressed in decimals of an inch, calculated for north latitude 40°." The third table "shows a

* See Guyot's Paper D, pp. 9 and 88.

small empirical correction, determined from accurate comparison of reduced readings and actual observations, to be applied to Table II." A formula is also given, which may be written $R = (N + N')\,Z$, in which N is the number from Table II., and N' that from Table III.

If we compare this formula with (iv.), it is evident that some correction to N is necessary, since R does not *vary as* Z. The correction should, however, depend on the reading of the barometer (β) as well as on Z and t; but the empirical correction N' is given without regard to β.

The constants and formula, on which Table II. is based, are not given; and the rate of variation of the numbers, with the pressure, seems to deviate more than it should, from Boyle's Law.

Lieut. Dunwoody's Tables have not, so far as I am aware, been anywhere brought into use. The results given by his Tables II. and III. do not, however, differ much at moderate altitudes from those given by Table A, as will be seen from the following examples:

EXAMPLES OF THE USE OF TABLE A.

Example (1).—At a station 815 ft. above the sea, the reading of the barometer being 29.112 in., the temperature of the air 46° Fahr., to find the reduced reading.

From Table A we find ${}_{800}N_{46} = 3.0047$, and the difference for 100 ft. $= 0.3819$.

Hence the reduction,

$$R = \left(3.0047 + \frac{15}{100} \times 0.3819\right) \times 0.29112 = 3.0620 \times 0.29112$$
$$= 0.891,$$

and the reduced reading is 30.003.

Guyot's tables D, XVI. and XIX.' used with formula (i.), each give, for this reduction, 0.876 in. Lieut. Dunwoody's tables (ii.) and (iii.) give 0.890.

Example (2).—At a station 1100 ft. above the sea, the reading of the barometer being 28 in., the temperature of the air 30° Fahr., to find the reduction to sea level.

Here ${}_{1000}N_{30} = 3.9071$, and the difference for 100 ft. is 0.3990,

hence $$R = (3.9071 + 0.3990) \times 0{\cdot}28 = 4.3061 \times 0.28$$
$$= 1.206.$$

Guyot's Tables D, XVI. and XIX.', if extended, used with formula (i.), would give in this case 1.179, and Lieut. Dunwoody's give 1.204.

The value of Table A does not, however, consist so much in supplying a basis for working out isolated examples, as in furnishing data, in a convenient form, for the calculation of tables of reduction to sea level, for individual stations. To construct these all that is necessary is, first, to obtain the numbers ${}_Z N_t$ for every second degree of temperature, the value assigned to Z being the height of the cistern of the barometer above the sea; and then, to multiply these numbers by $\frac{b}{100}$, and tabulate the values of the reduction so obtained for values of b, between convenient limits, and at larger or smaller intervals, according as the station is at a slight or considerable elevation above the sea. The products for any given temperature need not be obtained separately, but may be found, one from another, by continued addition, and the whole process may be very quickly performed with the aid of the Arithmometer of Thomas de Colmar, for use with which the table is specially adapted.

The time occupied in forming a table in this way, is less than one half of what is required if the formula of Laplace (iii. of this paper) be employed.

For stations more than 1100 ft. above the sea, Table B (from which Table A was deduced) may be employed. In this table the values of $\frac{100{,}000}{A_t}$ are given; so that if N_t is the number in the table for temperature t, formula iii. becomes

$$\log \frac{B}{\beta} = \frac{Z}{100{,}000} N_t ,$$

$$\text{or } \log B = \frac{Z}{100{,}000} N_t + \log \beta .$$

For isolated examples this form is sufficiently convenient; but, in constructing a table for any station, it is better to make $\beta = 100$. The formula then becomes

$$\log \left(100 + {}_Z N_t \right) = \frac{Z}{100{,}000} N_t + 2 ,$$

and the table may be calculated from the value of ${}_{Z}N_{t}$ in the same way as when Table A is employed.

A table for reducing the barometer to sea level is furnished from the Central Office, Toronto, to each station in connection with the Meteorological Service of the Dominion.

Formerly these were computed directly from formula iii. (using a slightly different constant from that given above.) The accompanying tables were recently calculated to diminish the labour of computation.

In Canada, no reduction for height is applied to the observed temperature of the air; as, although some correction might be of advantage, it is by no means certain that a correction, obtained from observations on a mountain, would be suited to a station on an elevated table-land. The correction, if it were applied, would, however, be very small at nearly all our stations.

I hope to discuss, more fully, on some future occasion the question of the necessity for a correction to the observed temperature of the air in reducing barometric readings.

TABLE A.

Giving the value of $N_{Z\ t}$ for various temperatures and elevations, and the difference for an additional 100 feet at each height.

Temperature.	100 Feet.		200 Feet.		300 Feet.		400 Feet.		500 Feet.		Temperature.
	$N_{Z\ t}$	Diff. for 100 ft.	$N_{Z\ t}$	Diff. for 100 ft.	$N_{Z\ t}$	Diff. for 100 ft.	$N_{Z\ t}$	Diff. for 100 ft.	$N_{Z\ t}$	Diff. for 100 ft.	
—40°	0.4553	.4573	0.9126	.4595	1.3721	.4615	1.8336	.4636	2.2972	.4658	—40°
—38	0.4529	.4549	0.9078	.4570	1.3648	.4590	1.8238	.4612	2.2850	.4632	—38
—36	0.4505	.4525	0.9030	.4546	1.3576	.4566	1.8142	.4587	2.2729	.4607	—36
—34	0.4482	.4501	0.8983	.4522	1.3505	.4542	1.8047	.4562	2.2609	.4583	—34
—32	0.4458	.4478	0.8936	.4498	1.3434	.4519	1.7953	.4538	2.2491	.4558	—32
—30	0.4435	.4455	0.8890	.4475	1.3365	.4494	1.7859	.4515	2.2374	.4534	—30
—28	0.4412	.4432	0.8844	.4452	1.3296	.4471	1.7767	.4491	2.2258	.4510	—28
—26	0.4390	.4409	0.8799	.4428	1.3227	.4448	1.7675	.4468	2.2143	.4487	—26
—24	0.4368	.4386	0.8754	.4406	1.3160	.4425	1.7585	.4444	2.2029	.4464	—24
—22	0.4345	.4365	0.8710	.4383	1.3093	.4402	1.7495	.4422	2.1917	.4440	—22
—20	0.4324	.4342	0.8666	.4361	1.3027	.4380	1.7407	.4398	2.1805	.4418	—20
—18	0.4302	.4320	0.8622	.4339	1.2961	.4358	1.7319	.4376	2.1695	.4395	—18
—16	0.4280	.4299	0.8579	.4317	1.2896	.4336	1.7232	.4354	2.1586	.4373	—16
—14	0.4259	.4277	0.8536	.4296	1.2832	.4314	1.7146	.4332	2.1478	.4351	—14
—12	0.4238	.4256	0.8494	.4274	1.2768	.4293	1.7061	.4310	2.1371	.4329	—12
—10	0.4217	.4235	0.8452	.4253	1.2705	.4271	1.6976	.4289	2.1265	.4307	—10
— 8	0.4197	.4214	0.8411	.4232	1.2643	.4250	1.6893	.4267	2.1160	.4286	— 8
— 6	0.4176	.4194	0.8370	.4211	1.2581	.4229	1.6810	.4247	2.1057	.4264	— 6
— 4	0.4156	.4173	0.8329	.4191	1.2520	.4208	1.6728	.4226	2.0954	.4243	— 4
— 2	0.4136	.4153	0.8289	.4171	1.2460	.4187	1.6647	.4205	2.0852	.4222	— 2
0	0.4116	.4133	0.8249	.4151	1.2400	.4167	1.6567	.4184	2.0751	.4202	0
2	0.4097	.4113	0.8210	.4130	1.2340	.4147	1.6487	.4164	2.0651	.4182	2
4	0.4077	.4094	0.8171	.4110	1.2281	.4128	1.6409	.4144	2.0553	.4160	4
6	0.4058	.4074	0.8132	.4091	1.2223	.4107	1.6330	.4125	2.0455	.4141	6
8	0.4039	.4055	0.8094	.4071	1.2165	.4088	1.6253	.4105	2.0358	.4121	8
10	0.4020	.4036	0.8056	.4052	1.2108	.4069	1.6177	.4085	2.0262	.4101	10
12	0.4001	.4017	0.8018	.4033	1.2051	.4050	1.6101	.4065	2.0166	.4082	12
14	0.3982	.3999	0.7981	.4014	1.1995	.4031	1.6026	.4046	2.0072	.4063	14
16	0.3964	.3980	0.7944	.3996	1.1940	.4011	1.5951	.4028	1.9979	.4043	16
18	0.3946	.3961	0.7907	.3978	1.1885	.3992	1.5877	.4009	1.9886	.4025	18
20	0.3928	.3943	0.7871	.3959	1.1830	.3974	1.5804	.3990	1.9794	.4006	20
22	0.3910	.3925	0.7835	.3941	1.1776	.3956	1.5732	.3972	1.9704	.3987	22
24	0.3892	.3908	0.7800	.3922	1.1722	.3938	1.5660	.3954	1.9614	.3968	24
26	0.3875	.3889	0.7764	.3905	1.1669	.3920	1.5589	.3935	1.9524	.3951	26
28	0.3857	.3872	0.7729	.3888	1.1617	.3902	1.5519	.3917	1.9436	.3932	28

TABLE A.—*Continued.*

Temperature.	600 Feet.		700 Feet.		800 Feet.		900 Feet.		1000 Feet.		Temperature.
	$\frac{N}{Z}t$	Diff. for 100 ft.	$\frac{N}{Z}t$	Diff. for 100 ft.	$\frac{N}{Z}t$	Diff. for 100 ft.	$\frac{N}{Z}t$	Diff. for 100 ft.	$\frac{N}{Z}t$	Diff. for 100 ft.	
—40°	2.7630	.4678	3.2308	.4700	3.7008	.4721	4.1729	.4743	4.6472	.4764	—40°
—38	2.7482	.4653	3.2135	.4675	3.6810	.4695	4.1505	.4717	4.6222	.4738	—38
—36	2.7336	.4629	3.1965	.4649	3.6614	.4670	4.1284	.4690	4.5974	.4713	—36
—34	2.7192	.4603	3.1795	.4624	3.6419	.4645	4.1064	.4666	4.5730	.4686	—34
—32	2.7049	.4579	3.1628	.4599	3.6227	.4620	4.0847	.4640	4.5487	.4661	—32
—30	2.6908	.4555	3.1463	.4574	3.6037	.4595	4.0632	.4616	4.5248	.4636	—30
—28	2.6768	.4531	3.1299	.4550	3.5849	.4571	4.0420	.4591	4.5011	.4611	—28
—26	2.6630	.4507	3.1137	.4526	3.5663	.4546	4.0209	.4567	4.4776	.4586	—26
—24	2.6493	.4483	3.0976	.4503	3.5479	.4522	4.0001	.4543	4.4544	.4562	—24
—22	2.6357	.4460	3.0817	.4480	3.5297	.4498	3.9795	.4519	4.4314	.4538	—22
—20	2.6223	.4437	3.0660	.4456	3.5116	.4475	3.9591	.4495	4.4086	.4514	—20
—18	2.6090	.4414	3.0504	.4433	3.4937	.4453	3.9390	.4471	4.3861	.4490	—18
—16	2.5959	.4391	3.0350	.4411	3.4761	.4429	3.9190	.4448	4.3638	.4467	—16
—14	2.5829	.4369	3.0198	.4388	3.4586	.4406	3.8992	.4425	4.3417	.4444	—14
—12	2.5700	.4347	3.0047	.4365	3.4412	.4384	3.8796	.4403	4.3199	.4421	—12
—10	2.5572	.4325	2.9897	.4344	3.4241	.4362	3.8603	.4380	4.2983	.4398	—10
— 8	2.5446	.4303	2.9749	.4322	3.4071	.4340	3.8411	.4358	4.2769	.4376	— 8
— 6	2.5321	.4282	2.9603	.4300	3.3903	.4318	3.8221	.4336	4.2557	.4354	— 6
— 4	2.5197	.4261	2.9458	.4278	3.3736	.4297	3.8033	.4314	4.2347	.4332	— 4
— 2	2.5074	.4240	2.9314	.4257	3.3571	.4275	3.7846	.4293	4.2139	.4310	— 2
0	2.4953	.4219	2.9172	.4236	3.3408	.4254	3.7662	.4271	4.1933	.4289	0
2	2.4833	.4198	2.9031	.4215	3.3246	4233	3.7479	.4250	4.1729	.4268	2
4	2.4713	.4178	2.8891	.4195	3.3086	.4212	3.7298	.4230	4.1528	.4246	4
6	2.4596	.4157	2.8753	.4175	3.2928	.4191	3.7119	.4209	4.1328	.4225	6
8	2.4479	.4137	2.8616	.4153	3.2769	.4173	3.6942	.4188	4.1130	.4205	8
10	2.4363	.4118	2.8481	.4134	3.2615	.4151	3.6766	.4168	4.0934	.4184	10
12	2.4248	.4098	2.8346	.4115	3.2461	.4131	3.6592	.4147	4.0739	.4165	12
14	2.4135	.4078	2.8213	.4095	3.2308	.4111	3.6419	.4128	4.0547	.4144	14
16	2.4022	.4059	2.8081	.4076	3.2157	.4092	3.6249	.4107	4.0356	.4125	16
18	2.3911	.4040	2.7951	.4056	3.2007	.4072	3.6079	.4089	4.0168	.4104	18
20	2.3800	.4021	2.7821	.4038	3.1859	.4053	3.5912	.4069	3.9981	.4085	20
22	2.3691	.4002	2.7693	.4019	3.1712	.4034	3.5746	.4049	3.9795	.4066	22
24	2.3582	.3984	2.7566	.4000	3.1566	.4015	3.5581	.4031	3.9612	.4046	24
26	2.3475	.3965	2.7440	.3981	3.1421	.3997	3.5418	.4012	3.9430	.4027	26
28	2.3368	.3948	2.7316	.3962	3.1278	.3978	3.5256	.3994	3.9250	.4008	28

TABLE A.—*Continued.*

Temperature.	100 Feet.		200 Feet.		300 Feet.		400 Feet.		500 Feet.		Temperature.
	N $Z\ t$	Diff. for 100 ft.	N $Z\ t$	Diff. for 100 ft.	N $Z\ t$	Diff. for 100 ft.	N $Z\ t$	Diff. for 100 ft.	N $Z\ t$	Diff. for 100 ft.	
30°	0.3840	.3855	0.7695	.3869	1.1564	.3885	1.5449	.3899	1.9348	.3915	30°
32	0.3823	.3838	0.7661	.3852	1.1513	.3867	1.5380	.3882	1.9262	.3896	32
34	0.3806	.3820	0.7626	.3836	1.1462	.3849	1.5311	.3864	1.9175	.3879	34
36	0.3789	.3804	0.7593	.3818	1.1411	.3832	1.5243	.3847	1.9090	.3862	36
38	0.3773	.3786	0.7559	.3801	1.1360	.3816	1.5176	.3830	1.9006	.3844	38
40	0.3756	.3770	0.7526	.3785	1.1311	.3798	1.5109	.3813	1.8922	.3827	40
42	0.3740	.3753	0.7493	.3768	1.1261	.3782	1.5043	.3796	1.8839	.3810	42
44	0.3723	.3738	0.7461	.3751	1.1212	.3765	1.4977	.3780	1.8757	.3793	44
46	0.3707	.3722	0.7429	.3734	1.1163	.3749	1.4912	.3763	1.8675	.3777	46
48	0.3691	.3706	0.7397	.3718	1.1115	.3733	1.4848	.3746	1.8594	.3760	48
50	0.3676	.3689	0.7365	.3703	1.1068	.3716	1.4784	.3730	1.8514	.3744	50
52	0.3660	.3673	0.7333	.3687	1.1020	.3700	1.4720	.3714	1.8434	.3728	52
54	0.3644	.3658	0.7302	.3671	1.0973	.3685	1.4658	.3698	1.8356	.3711	54
56	0.3629	.3642	0.7271	.3656	1.0927	.3668	1.4595	.3682	1.8277	.3696	56
58	0.3614	.3627	0.7241	.3640	1.0881	.3653	1.4534	.3666	1.8200	.3679	58
60	0.3599	.3611	0.7210	.3625	1.0835	.3637	1.4472	.3651	1.8123	.3664	60
62	0.3584	.3596	0.7180	.3609	1.0789	.3623	1.4412	.3635	1.8047	.3648	62
64	0.3569	.3581	0.7150	.3594	1.0744	.3607	1.4351	.3620	1.7971	.3633	64
66	0.3554	.3566	0.7120	.3580	1.0700	.3592	1.4292	.3604	1.7896	.3618	66
68	0.3539	.3552	0.7091	.3564	1.0655	.3577	1.4232	.3590	1.7822	.3602	68
70	0.3525	.3537	0.7062	.3550	1.0612	.3562	1.4174	.3574	1.7748	.3588	70
72	0.3510	.3523	0.7033	.3535	1.0568	.3547	1.4115	.3560	1.7675	.3573	72
74	0.3496	.3508	0.7004	.3521	1.0525	.3533	1.4058	.3545	1.7603	.3557	74
76	0.3482	.3494	0.6976	.3506	1.0482	.3518	1.4000	.3531	1.7531	.3543	76
78	0.3468	.3480	0.6948	.3492	1.0440	.3504	1.3944	.3516	1.7460	.3529	78
80	0.3454	.3466	0.6920	.3477	1.0397	.3490	1.3887	.3502	1.7389	.3514	80
82	0.3440	.3452	0.6892	.3464	1.0356	.3475	1.3831	.3488	1.7319	.3499	82
84	0.3426	.3438	0.6864	.3450	1.0314	.3462	1.3776	.3473	1.7249	.3486	84
86	0.3413	.3424	0.6837	.3436	1.0273	.3448	1.3721	.3459	1.7180	.3471	86
88	0.3399	.3411	0.6810	.3422	1.0232	.3434	1.3666	.3446	1.7112	.3457	88
90	0.3386	.3397	0.6783	.3409	1.0192	.3420	1.3612	.3432	1.7044	.3443	90
92	0.3372	.3384	0.6756	.3395	1.0151	.3407	1.3558	.3418	1.6976	.3430	92
94	0.3359	.3371	0.6730	.3382	1.0112	.3393	1.3505	.3404	1.6909	.3416	94
96	0.3346	.3358	0.6704	.3368	1.0072	.3380	1.3452	.3391	1.6843	.3403	96
98	0.3333	.3344	0.6677	.3356	1.0033	.3366	1.3399	.3378	1.6777	.3389	98
100	0.3320	.3332	0.6652	.3342	0.9994	.3353	1.3347	.3365	1.6712	.3376	100

TABLE A.—*Continued.*

Temperature.	600 Feet.		700 Feet.		800 Feet.		900 Feet.		1000 Feet.		Temperature.
	$\frac{N}{Z\ t}$	Diff. for 100 ft.	$\frac{N}{Z\ t}$	Diff. for 100 ft.	$\frac{N}{Z\ t}$	Diff. for 100 ft.	$\frac{N}{Z\ t}$	Diff. for 100 ft.	$\frac{N}{Z\ t}$	Diff. for 100 ft.	
30°	2.3263	.3929	2.7192	.3945	3.1137	.3959	3.5096	.3975	3.9071	.3990	30°
32	2.3158	.3912	2.7070	.3926	3.0996	.3941	3.4937	.3957	3.8894	.3972	32
34	2.3054	.3894	2.6948	.3909	3.0857	.3923	3.4780	.3939	3.8719	.3953	34
36	2.2952	.3876	2.6828	.3891	3.0719	.3905	3.4624	.3921	3.8545	.3935	36
38	2.2850	.3859	2.6709	.3873	3.0582	.3888	3.4470	.3903	3.8373	.3917	38
40	2.2749	.3842	2.6591	.3855	3.0446	.3871	3.4317	.3885	3.8202	.3899	40
42	2.2649	.3824	2.6473	.3839	3.0312	.3853	3.4165	.3868	3.8033	.3882	42
44	2.2550	.3807	2.6357	.3822	3.0179	.3836	3.4015	.3850	3.7865	.3864	44
46	2.2452	.3790	2.6242	.3805	3.0047	.3819	3.3866	.3833	3.7699	.3847	46
48	2.2354	.3774	2.6128	.3788	2.9916	.3802	3.3718	.3816	3.7534	.3830	48
50	2.2258	.3757	2.6015	.3771	2 9786	.3785	3.3571	.3799	3.7370	.3813	50
52	2.2162	.3741	2.5903	.3755	2.9658	.3768	3.3426	.3783	3.7209	.3796	52
54	2.2067	.3725	2.5792	.3738	2.9530	.3752	3.3282	.3766	3.7048	.3779	54
56	2.1973	.3709	2.5682	.3722	2.9404	.3735	3.3139	.3750	3.6889	.3763	56
58	2.1879	.3693	2.5572	.3706	2.9278	.3720	3.2998	.3733	3.6731	.3747	58
60	2.1787	.3677	2.5464	.3690	2.9154	.3704	3.2858	.3717	3.6575	.3730	60
62	2.1695	.3661	2.5356	.3675	2.9031	.3688	3.2719	.3700	3.6419	.3715	62
64	2.1604	.3646	2.5250	.3659	2.8909	.3672	3.2581	.3685	3.6266	.3698	64
66	2.1514	.3630	2.5144	.3644	2.8788	.3656	3.2444	.3669	3.6113	.3682	66
68	2.1424	.3616	2.5040	.3627	2.8667	.3641	3.2308	.3654	3.5962	.3666	68
70	2.1336	.3600	2.4936	.3613	2.8549	.3625	3.2174	.3638	3.5812	.3651	70
72	2.1248	.3585	2.4833	.3597	2.8430	.3610	3.2040	.3623	3.5663	.3636	72
74	2.1160	.3570	2.4730	.3583	2.8313	.3595	3.1908	.3608	3.5516	.3620	74
76	2.1074	.3555	2.4629	.3568	2.8197	.3580	3.1777	.3592	3.5369	.3605	76
78	2.0989	.3540	2.4529	.3552	2.8081	.3566	3.1647	.3577	3.5224	.3590	78
80	2.0903	.3526	2.4429	.3538	2.7967	.3551	3.1518	.3562	3.5080	.3575	80
82	2.0818	.3512	2.4330	.3524	2.7854	.3535	3.1389	.3548	3.4937	.3561	82
84	2.0735	.3497	2.4232	.3509	2.7741	.3522	3.1263	.3533	3.4796	.3545	84
86	2.0651	.3484	2.4135	.3495	2.7630	.3507	3.1137	.3518	3.4655	.3531	86
88	2.0569	.3469	2.4038	.3481	2.7519	.3493	3.1012	.3504	3.4516	.3514	88
90	2.0487	.3455	2.3942	.3467	2.7409	.3479	3.0888	.3490	3.4378	.3502	90
92	2.0406	.3441	2.3847	.3453	2.7300	.3465	3.0765	.3476	3.4241	.3488	92
94	2.0325	.3428	2 3753	.3439	2.7192	.3451	3.0643	.3462	3.4105	.3474	94
96	2.0246	.3414	2.3660	.3425	2.7085	.3437	3.0522	.3448	3.3970	.3460	96
98	2.0166	.3401	2.3567	.3411	2.6978	.3424	4.0402	.3434	3.3836	.3446	68
100	2.0088	.3387	2.3475	.3398	2.6873	.3409	3.0282	.3421	3.3703	.3432	100

TABLE B,

Giving the value of $\frac{100,000}{A_t}$ for vaiious values of t, the value of A_t being

$$60345.51 \left\{ 1 + \frac{t-32}{450} \right\}$$

Temperature.	$\frac{100,000}{A_t}$	Temperature.	$\frac{100,000}{A_t}$	Temperature.	$\frac{100,000}{A_t}$
—40°	1.972767	8°	1.750483	56°	1.573219
—38	1.962384	10	1.742303	58	1.566609
—36	1.952110	12	1.734200	60	1.560054
—34	1.941942	14	1.726171	62	1.553554
—32	1.931880	16	1.718216	64	1.547108
—30	1.921922	18	1.710335	66	1.540715
—28	1.912066	20	1.702525	68	1.534374
—26	1.902311	22	1.694786	70	1.528086
—24	1.892654	24	1.687117	72	1.521849
—22	1.883096	26	1.679518	74	1.515662
—20	1.873633	28	1.671986	76	1.509526
—18	1.864265	30	1.664522	78	1.503439
—16	1.854990	32	1.657124	80	1.497401
—14	1.845807	34	1.649792	82	1.491412
—12	1.836714	36	1.642524	84	1.485470
—10	1.827710	38	1.635320	86	1.479575
— 8	1.818795	40	1.628179	88	1.473727
— 6	1.809966	42	1.621100	90	1.467925
— 4	1.801222	44	1.614082	92	1.462168
— 2	1.792562	46	1.607125	94	1.456457
0	1.783985	48	1.600227	96	1.450790
2	1.775490	50	1.593389	98	1.445166
4	1.767075	52	1.586608	100	1.439587
6	1.758740	54	1.579885		